First 4 Riesel conjectures

## Definition

For the original Riesel problem, it is finding and proving the smallest *k* such that *k*×*bn*-1 is not prime for all integers *n* ≥ 1 and GCD(*k*-1, *b*-1)=1.

### **Extended definiton**

Finding and proving the smallest *k* such that (*k*×*bn*-1)/GCD(*k*-1, *b*-1) is not prime for all integers *n* ≥ 1.

### **Notes**

All *n* must be >= 1.

*k*-values that make a full covering set with all or partial algebraic factors are excluded from the conjectures.

*k*-values that are a multiple of base (*b*) and where (*k*-1)/gcd(*k*-1,*b*-1) is not prime are included in the conjectures but excluded from testing.

Such *k*-values will have the same prime as *k* / *b*.

## Table

| **Base** | **Conjectured first 4 Riesel *k*** | ***k*'s that make a full covering set with all or partial algebraic factors** | **Remaining *k* to find prime**  **(*n* testing limit)** | **Top 10 *k*'s with largest first primes: *k* (*n*)**  **(sorted by *n* only)** | **Comments** |
| --- | --- | --- | --- | --- | --- |
| **4** | 361, 919, 1114, 1444 | All k = m^2 for all n;  factors to:  (m\*2^n - 1) \*  (m\*2^n + 1) | none - proven | 659 (400258)  1211 (12621)  751 (6615)  674 (5838)  1159 (5628)  106 (4553)  1189 (3404)  1171 (2855)  373 (2508)  1103 (2203) | k = 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 11^2, 12^2, 13^2, 14^2, 15^2, 16^2, etc. (except 19^2) proven composite by full algebraic factors. |
| **5** | 13, 17, 37, 41 |  | none - proven | 34 (163)  38 (28)  32 (8)  31 (5)  35 (4)  26 (4)  23 (4)  2 (4)  1 (3)  29 (2) |  |
| **7** | 457, 1291, 3199, 3313 |  | 679, 691, 717, 859, 919, 1031, 1179, 1459, 1651, 1679, 1693, 1747, 1811, 1831, 1873, 1979, 2011, 2131, 2137, 2253, 2311, 2623, 2673, 2791, 2797, 2839, 2887, 3139, 3181, 3217, 3307 (k = 1 mod 6 at n=3K, other k at n=15K) | 197 (181761)  367 (15118)  313 (5907)  1793 (5839)  159 (4896)  1469 (4669)  429 (3815)  3033 (2819)  2473 (2779)  2493 (2567) |  |
| **8** | 14, 112, 116, 148 | All k = m^3 for all n;  factors to:  (m\*2^n - 1) \*  (m^2\*4^n + m\*2^n + 1) | none - proven | 74 (2632)  37 (851)  142 (463)  73 (389)  92 (314)  127 (139)  104 (96)  47 (26)  84 (24)  43 (21) | k = 1, 8, 27, 64, and 125 proven composite by full algebraic factors. |
| **9** | 41, 49, 74, 121 | All k = m^2 for all n;  factors to:  (m\*3^n - 1) \*  (m\*3^n + 1) | none - proven | 119 (4486)  53 (536)  71 (23)  87 (15)  94 (12)  11 (11)  107 (9)  89 (8)  24 (8)  14 (8) | k = 1, 4, 9, 16, 25, 36, 64, 81, and 100 proven composite by full algebraic factors. |
| **10** | 334, 1585, 1882, 3340 | k = 343:  n = = 1 mod 3:  factor of 3  n = = 2 mod 3:  factor of 37  n = = 0 mod 3:  let n=3q; factors to:  (7\*10^q - 1) \* [49\*10^(2q) + 7\*10^q + 1] | 2452 (554.7K) | 1935 (51836)  1803 (45882)  1231 (37398)  1343 (29711)  505 (18470)  450 (11958)  3112 (3292)  1198 (2890)  2276 (2726)  2333 (2113) |  |
| **11** | 5, 7, 17, 19 |  | none - proven | 1 (17)  9 (5)  16 (3)  15 (2)  14 (2)  8 (2)  3 (2)  2 (2)  18 (1)  13 (1) |  |
| **12** | 376, 742, 1288, 1364 | (Condition 1):  All k where k = m^2  and m = = 5 or 8 mod 13:  for even n let k = m^2  and let n = 2\*q; factors to:  (m\*12^q - 1) \*  (m\*12^q + 1)  odd n:  factor of 13  (Condition 2):  All k where k = 3\*m^2  and m = = 3 or 10 mod 13:  even n:  factor of 13  for odd n let k = 3\*m^2  and let n=2\*q-1; factors to:  [m\*2^(2q-1)\*3^q - 1] \*  [m\*2^(2q-1)\*3^q + 1] | none - proven | 1132 (28717)  1037 (6281)  298 (1676)  1119 (1351)  1262 (1017)  534 (781)  844 (744)  943 (676)  647 (545)  831 (318) | k = 25, 64, 324, 441, 961, and 1156 proven composite by condition 1.  k = 27, 300, and 768 proven composite by condition 2. |
| **13** | 29, 41, 69, 85 |  | none - proven | 43 (77)  76 (34)  52 (18)  25 (15)  28 (14)  20 (10)  34 (8)  72 (6)  47 (6)  42 (6) |  |
| **14** | 4, 11, 19, 26 | All k where k = m^2  and m = = 2 or 3 mod 5:  for even n let k = m^2  and let n = 2\*q; factors to:  (m\*14^q - 1) \*  (m\*14^q + 1)  odd n:  factor of 5 | none - proven | 5 (19698)  2 (4)  1 (3)  24 (2)  23 (2)  20 (2)  17 (2)  15 (2)  8 (2)  25 (1) | k = 9 proven composite by partial algebraic factors. |
| **16** | 100, 172, 211, 295 | All k = m^2 for all n;  factors to:  (m\*4^n - 1) \*  (m\*4^n + 1) | none - proven | 74 (638)  137 (545)  178 (276)  148 (266)  273 (245)  219 (103)  152 (98)  235 (68)  203 (58)  263 (45) | k = 1, 4, 9, 16, 25, 36, 49, 64, 81, 121, 144, 169, 196, 225, 256, and 289 proven composite by full algebraic factors. |
| **17** | 49, 59, 65, 86 |  | none - proven | 44 (6488)  29 (4904)  13 (1123)  36 (243)  10 (117)  26 (110)  5 (60)  11 (46)  58 (35)  46 (25) |  |
| **18** | 246, 664, 723, 837 |  | 533, 597 (both at n=3K) | 324 (25665)  628 (2213)  474 (1316)  457 (951)  501 (481)  337 (452)  151 (418)  811 (409)  711 (354)  261 (347) |  |
| **19** | 9, 11, 29, 31 |  | none - proven | 23 (108)  1 (19)  18 (6)  14 (6)  17 (5)  27 (4)  26 (3)  24 (2)  15 (2)  10 (2) |  |
| **20** | 8, 13, 29, 34 |  | none - proven | 17 (22)  15 (21)  2 (10)  16 (9)  11 (8)  14 (6)  28 (3)  1 (3)  32 (2)  27 (2) |  |
| **21** | 45, 65, 133, 153 |  | none - proven | 64 (2867)  131 (222)  101 (144)  47 (98)  29 (98)  84 (88)  142 (48)  109 (48)  61 (36)  77 (20) |  |
| **23** | 5, 7, 17, 19 |  | none - proven | 14 (52)  3 (6)  2 (6)  4 (5)  1 (5)  18 (2)  15 (2)  12 (2)  11 (2)  8 (2) |  |
| **25** | 105, 129, 211, 313 | All k = m^2 for all n;  factors to:  (m\*5^n - 1) \*  (m\*5^n + 1) | 181, 235 (both at n=3K) | 86 (1029)  268 (237)  177 (87)  274 (83)  170 (81)  137 (76)  265 (54)  58 (26)  272 (24)  130 (24) | k = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, and 289 proven composite by full algebraic factors. |
| **26** | 149, 334, 1892, 1987 |  | 178, 191, 223, 254, 284, 317, 355, 368, 380, 454, 562, 628, 729, 874, 892, 898, 926, 1061, 1147, 1153, 1157, 1184, 1189, 1204, 1208, 1214, 1279, 1312, 1375, 1376, 1541, 1549, 1657, 1736, 1774, 1852, 1901, 1930, 1934, 1945, 1963 (all at n=3K) | 115 (520277)  32 (9812)  1094 (2586)  1186 (2541)  1237 (2277)  913 (1913)  1514 (1638)  121 (1509)  1704 (1486)  410 (1308) |  |
| **27** | 13, 15, 41, 43 | All k = m^3 for all n;  factors to:  (m\*3^n - 1) \*  (m^2\*9^n + m\*3^n + 1) | none - proven | 23 (3742)  9 (23)  29 (13)  11 (10)  39 (8)  34 (8)  20 (8)  19 (8)  33 (7)  42 (5) | k = 1, 8, and 27 proven composite by full algebraic factors. |
| **29** | 4, 9, 11, 13 |  | none - proven | 2 (136)  8 (38)  1 (5)  10 (3)  5 (2)  12 (1)  7 (1)  6 (1)  3 (1) |  |
| **31** | 145, 265, 443, 493 |  | 5, 19, 51, 73, 97, 179, 191, 223, 235, 239, 247, 259, 274, 415, 421, 463, 467, 487, 489 (all at n=3K) | 401 (2977)  123 (1872)  313 (1605)  209 (1589)  214 (1143)  124 (1116)  113 (643)  49 (637)  115 (464)  391 (378) |  |
| **32** | 10, 23, 43, 56 | All k = m^5 for all n;  factors to:  (m\*2^n - 1) \*  (m^4\*16^n + m^3\*8^n + m^2\*4^n + m\*2^n + 1) | 29 (2M) | 37 (6425)  13 (159)  44 (72)  26 (58)  54 (24)  39 (21)  47 (14)  3 (11)  53 (10)  42 (10) | k = 1 and 32 proven composite by full algebraic factors. |
| **33** | 545, 577, 764, 1633 | (Condition 1):  All k where k = m^2  and m = = 4 or 13 mod 17:  for even n let k = m^2  and let n = 2\*q; factors to:  (m\*33^q - 1) \*  (m\*33^q + 1)  odd n:  factor of 17  (Condition 2):  All k where k = 33\*m^2  and m = = 4 or 13 mod 17:  [Reverse condition 1]  (Condition 3):  All k where k = m^2  and m = = 15 or 17 mod 32:  for even n let k = m^2  and let n = 2\*q; factors to:  (m\*33^q - 1) \*  (m\*33^q + 1)  odd n:  factor of 2 | 257, 339, 817, 851, 951, 1123, 1240 (k = 257 and 339 at n=12K, other k at n=3K) | 732 (19011)  186 (16770)  254 (3112)  562 (3087)  1408 (2920)  1157 (2647)  142 (2568)  1327 (1691)  1582 (1651)  370 (1628) | k = 16, 169, 441, 900, and 1444 proven composite by condition 1.  k = 528 proven composite by condition 2.  k = 225 and 289 proven composite by condition 3. |
| **34** | 6, 29, 41, 64 |  | none - proven | 27 (3086)  31 (75)  44 (36)  59 (34)  57 (23)  33 (15)  1 (13)  20 (10)  22 (5)  21 (5) |  |
| **35** | 5, 7, 17, 19 |  | none - proven | 1 (313)  3 (6)  2 (6)  16 (5)  14 (4)  8 (4)  15 (2)  11 (2)  6 (2)  18 (1) |  |
| **37** | 29, 77, 113, 163 |  | 33, 149 (k = 33 at n=10K, k = 149 at n=3K) | 81 (7683)  162 (1450)  56 (1158)  5 (900)  101 (574)  92 (313)  109 (188)  130 (146)  100 (99)  141 (74) |  |
| **38** | 13, 14, 25, 53 |  | 44 (3K) | 37 (136211)  22 (1579)  11 (766)  9 (43)  19 (41)  41 (26)  27 (23)  50 (16)  31 (15)  33 (9) |  |
| **39** | 9, 11, 29, 31 | All k where k = m^2  and m = = 2 or 3 mod 5:  for even n let k = m^2  and let n = 2\*q; factors to:  (m\*39^q - 1) \*  (m\*39^q + 1)  odd n:  factor of 5 | none - proven | 1 (349)  14 (100)  24 (94)  16 (35)  30 (8)  15 (4)  13 (3)  27 (2)  23 (2)  19 (2) | k = 4 proven composite by partial algebraic factors. |
| **41** | 8, 13, 17, 25 |  | none - proven | 14 (212)  7 (153)  5 (10)  23 (6)  18 (4)  11 (4)  22 (3)  1 (3)  20 (2)  6 (2) |  |
| **43** | 21, 23, 65, 67 |  | 13, 55 (k = 13 at n=50K, k = 55 at n=3K) | 53 (301)  4 (279)  35 (204)  12 (203)  61 (87)  17 (79)  59 (76)  39 (40)  31 (38)  3 (24) |  |
| **44** | 4, 11, 19, 26 |  | none - proven | 23 (18)  8 (16)  24 (14)  20 (12)  16 (9)  12 (5)  1 (5)  2 (4)  25 (3)  21 (3) |  |
| **45** | 93, 137, 277, 321 |  | 197, 257 (both at n=3K) | 24 (153355)  53 (582)  286 (211)  205 (174)  70 (167)  313 (165)  106 (161)  102 (160)  29 (146)  204 (141) |  |
| **47** | 5, 7, 13, 14 |  | none - proven | 4 (1555)  1 (127)  10 (51)  8 (32)  2 (4)  11 (2)  3 (2)  12 (1)  9 (1)  6 (1) |  |
| **49** | 81, 129, 229, 241 | All k = m^2 for all n;  factors to:  (m\*7^n - 1) \*  (m\*7^n + 1) | 82 (10K) | 230 (24824)  194 (2530)  159 (2448)  139 (234)  79 (212)  115 (128)  216 (125)  44 (122)  106 (69)  209 (64) | k = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, and 225 proven composite by full algebraic factors. |
| **50** | 16, 35, 67, 86 | (Condition 1):  All k where k = m^2  and m = = 4 or 13 mod 17:  for even n let k = m^2  and let n = 2\*q; factors to:  (m\*50^q - 1) \*  (m\*50^q + 1)  odd n:  factor of 17  (Condition 2):  All k where k = 2\*m^2  and m = = 3 or 14 mod 17:  even n:  factor of 17  for odd n let k = 2\*m^2  and let n=2\*q-1; factors to:  [m\*5^(2q-1)\*2^q - 1] \*  [m\*5^(2q-1)\*2^q + 1] | 37, 68 (k = 37 at n=121K, k = 68 at n=3K) | 76 (1049)  14 (66)  49 (25)  73 (19)  52 (19)  13 (19)  84 (12)  5 (12)  75 (11)  44 (8) | No k's proven composite by condition 1.  k = 18 proven composite by condition 2. |
| **51** | 25, 27, 77, 79 | All k where k = m^2  and m = = 5 or 8 mod 13:  for even n let k = m^2  and let n = 2\*q; factors to:  (m\*51^q - 1) \*  (m\*51^q + 1)  odd n:  factor of 13 | none - proven | 1 (4229)  23 (96)  47 (40)  74 (23)  45 (21)  75 (10)  53 (9)  62 (8)  39 (8)  3 (8) | k = 64 proven composite by partial algebraic factors. |
| **53** | 13, 17, 37, 41 |  | none - proven | 33 (1877)  22 (211)  12 (71)  10 (71)  2 (44)  29 (40)  26 (30)  38 (24)  14 (20)  16 (15) |  |
| **54** | 21, 34, 76, 89 | (Condition 1):  All k where k = m^2  and m = = 2 or 3 mod 5:  for even n let k = m^2  and let n = 2\*q; factors to:  (m\*54^q - 1) \*  (m\*54^q + 1)  odd n:  factor of 5  (Condition 2):  All k where k = 6\*m^2  and m = = 1 or 4 mod 5:  even n:  factor of 5  for odd n let k = 6\*m^2  and let n=2\*q-1; factors to:  [m\*2^q\*3^(3q-1) - 1] \*  [m\*2^q\*3^(3q-1) + 1] | 45 (3K) | 32 (1044)  87 (310)  23 (267)  59 (200)  82 (48)  46 (43)  26 (37)  79 (34)  44 (22)  72 (19) | k = 4, 9, 49, and 64 proven composite by condition 1.  k = 6 proven composite by condition 2. |
| **55** | 13, 15, 41, 43 |  | none - proven | 3 (76)  22 (21)  1 (17)  27 (8)  11 (8)  23 (6)  20 (6)  39 (4)  19 (4)  9 (3) |  |
| **56** | 20, 37, 77, 94 |  | 43 (3K) | 59 (276)  83 (238)  76 (211)  29 (108)  65 (66)  74 (64)  36 (45)  38 (38)  26 (32)  88 (29) |  |
| **57** | 144, 177, 233, 289 | All k where k = m^2  and m = = 3 or 5 mod 8:  for even n let k = m^2  and let n = 2\*q; factors to:  (m\*57^q - 1) \*  (m\*57^q + 1)  odd n:  factor of 2 | none - proven (with probable primes that have not been certified: k = 281) | 281 (5610)  242 (1188)  87 (242)  262 (241)  278 (184)  204 (163)  54 (157)  201 (138)  173 (112)  100 (109) | k = 9, 25, 121, and 169 proven composite by partial algebraic factors. |
| **59** | 4, 5, 7, 9 |  | none - proven | 3 (8)  1 (3)  8 (2)  2 (2)  6 (1) |  |
| **61** | 125, 185, 373, 433 |  | 37, 53, 100, 139, 165, 229, 313, 353, 365, 389, 421 (all at n=3K) | 198 (41855)  404 (18637)  13 (4134)  77 (3080)  131 (2464)  430 (2248)  10 (1552)  406 (1289)  156 (1049)  41 (755) |  |
| **62** | 8, 13, 29, 34 |  | 22, 26 (both at n=3K) | 14 (80)  3 (59)  28 (51)  4 (9)  17 (6)  19 (5)  20 (4)  31 (3)  1 (3)  33 (2) |  |
| **64** | 14, 51, 79, 116 | All k = m^2 for all n; factors to:  (m\*8^n - 1) \*  (m\*8^n + 1)  -or-  All k = m^3 for all n; factors to:  (m\*4^n - 1) \*  (m^2\*16^n + m\*4^n + 1) | none - proven | 24 (3020)  74 (1316)  106 (263)  92 (157)  99 (122)  69 (90)  104 (48)  85 (26)  47 (13)  84 (12) | k = 1, 4, 8, 9, 16, 25, 27, 36, 49, 64, 81, and 100 proven composite by full algebraic factors. |
| **81** | 74, 575, 657, 737 | All k = m^2 for all n;  factors to:  (m\*9^n - 1) \*  (m\*9^n + 1) | 123, 302, 477, 478, 630, 698, 716, 731 (all at n=3K) | 581 (2403)  119 (2243)  487 (1405)  366 (1388)  443 (995)  513 (873)  241 (600)  327 (492)  546 (429)  662 (403) | k = 1, 4, 9, 16, 25, 36, 49, 64, 81, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, and 729 proven composite by full algebraic factors. |
| **100** | 211, 235, 334, 750 | (Condition 1):  All k = m^2 for all n;  factors to:  (m\*10^n - 1) \*  (m\*10^n + 1)  (Condition 2):  k = 343:  n = = 1 mod 3:  factor of 37  n = = 2 mod 3:  factor of 3  n = = 0 mod 3:  let n=3q; factors to:  (7\*100^q - 1) \* [49\*100^(2q) + 7\*10^q + 1] | none - proven (with probable primes that have not been certified: k = 133, 469, and 505) | 653 (717513)  74 (44709)  505 (9235)  450 (5979)  133 (5496)  469 (4451)  302 (2132)  470 (1957)  630 (1691)  690 (1310) | k = 1, 4, 9, 16, 25, 36, 49, 64, 81, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, and 729 proven composite by condition 1.  k = 343 proven composite by condition 2. |
| **128** | 44, 59, 85, 86 | All k = m^7 for all n;  factors to:  (m\*2^n - 1) \*  (m^6\*64^n + m^5\*32^n + m^4\*16^n + m^3\*8^n + m^2\*4^n + m\*2^n + 1) | 46 (142.857K) | 29 (211192)  62 (44484)  23 (2118)  26 (1442)  74 (1128)  76 (759)  37 (699)  16 (459)  42 (246)  72 (124) |  |
| **256** | 100, 172, 211, 295 | All k = m^2 for all n;  factors to:  (m\*16^n - 1) \*  (m\*16^n + 1) | 191, 261, 286 (all at n=3K) | 242 (37762)  262 (1856)  282 (948)  219 (471)  247 (336)  74 (319)  47 (228)  42 (224)  274 (148)  92 (143) | k = 1, 4, 9, 16, 25, 36, 49, 64, 81, 121, 144, 169, 196, 225, 256, and 289 proven composite by full algebraic factors. |
| **512** | 14, 20, 37, 38 | All k = m^3 for all n;  factors to:  (m\*8^n - 1) \*  (m^2\*64^n + m\*8^n + 1) | none - proven | 26 (3290)  4 (2215)  13 (2119)  24 (1655)  32 (472)  31 (93)  34 (51)  33 (40)  35 (34)  19 (21) | k = 1, 8, and 27 proven composite by full algebraic factors. |
| **1024** | 81, 121, 124, 169 | All k = m^2 for all n; factors to:  (m\*32^n - 1) \*  (m\*32^n + 1)  -or-  All k = m^5 for all n;  factors to:  (m\*4^n - 1) \*  (m^4\*256^n + m^3\*64^n + m^2\*16^n + m\*4^n + 1) | 29, 31, 56, 61, 84, 91, 106, 109, 116, 136, 157, 166 (k = 29 at n=1M, other k at n=3K) | 74 (666084)  39 (4070)  43 (2290)  99 (1226)  13 (1167)  151 (639)  78 (424)  105 (281)  114 (276)  137 (218) | k = 1, 4, 9, 16, 25, 32, 36, 49, 64, 100, and 144 proven composite by full algebraic factors. |